Hw 2_ 2050B

1. Show that (-1) a = -a (that is LHS is the inverse of a) for any real number a.

2*. Show that the product of 0 with any real number a is 0 and that -(-a)= a (that is LHS is the inverse of -a with respect to addition) for any real number a. Show that (-1)^2 = 1 and
(-a)^2 = [(-1)a]^2=([-1]^2)(a^2) = a^2 for any a.
Hints: (1) (0) = 1 (0+0) = (1) (0) + (1)(0) and "cancellation law; -(-a) + (-a) = ((-1) +1) (-a) =0.
3. Show that the square a^2 is non-negative for any a.

4*. Let r be a real number and A be a bounded above, nonempty set of real numbers. Define the meaning that r: = sup A, the smallest (= the least) upper bound of A and complete the following sentences:
(i) If t < r then t < ...., for .... in A.
(ii) If t bigger than or equal to r then t is bigger than or equal to ...., for .... in A

$t \geq \sup A \implies t \geq a$, for (some / all ??) $a \in A$.

Do the corresponding question for inf B, the greatest lower bound of a bounded below nonempty set B of real numbers.

5* Let A be as in Q4 and let -A : = {-a: a belongs to A}. Show that -A is bounded below and inf -A = -sup A.

6 (i)Let A, B be bounded above nonempty subsets of real numbers and A+B = {a+b: a belongs to A, b belongs to B}. Show that A+B is also bounded above and sup(A+B) = sup A + sup B but that the equality

sup { f(x) + g(x) : x in D} = sup {f(x): x in D } + sup {g(x); x in D}

may fail, where D is a subset of R and f, g are real-valued functions on D such that {f(x): x in D} and {g(x): x inD} are bounded above.
6(ii)*. Do the corresponding question for inf in place of sup.